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Exact solution for a degenerate Anderson impurity in the U $\rightarrow \infty$ limit embedded into a correlated host*

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Abstract. We consider the one-dimensional $t-J$ model, which consists of electrons with spin S on a lattice with nearest neighbor hopping t constrained by the excluded multiple occupancy of the lattice sites and spin-exchange J between neighboring sites. The model is integrable at the supersymmetric point, $J = t$. Without spoiling the integrability we introduce an Anderson-like impurity of spin S (degenerate Anderson model in the $U \rightarrow \infty$ limit), which interacts with the correlated conduction states of the host. The lattice model is defined by the scattering matrices *via* the Quantum Inverse Scattering Method. We discuss the general form of the interaction Hamiltonian between the impurity and the itinerant electrons on the lattice and explicitly construct it in the continuum limit. The discrete Bethe ansatz equations diagonalizing the host with impurity are derived, and the thermodynamic Bethe ansatz equations are obtained using the string hypothesis for arbitrary band filling as a function of temperature and external magnetic field. The properties of the impurity depend on one coupling parameter related to the Kondo exchange coupling. The impurity can localize up to one itinerant electron and has in general mixed valent properties. Ground-state properties of the impurity, such as the energy, valence, magnetic susceptibility and the specific heat γ coefficient, are discussed. In the integer valent limit the model reduces to a Coqblin-Schrieffer impurity.

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1 Introduction

Impurities play an important role in strongly correlated electron systems, specially in one-dimension (1D), where even a small amount of defects may change the properties drastically. Interactions in the host are particularly important in 1D, where the system changes from normal Fermi liquid to Luttinger liquid [1]. Effects of the interactions on the properties of an impurity have been investigated by bosonization, renormalization-groups, poor man's scaling, boundary conformal field theory [2], and the Bethe *Ansatz* [3–7].

An impurity introduced into an integrable host usually destroys the integrability. The interaction between the host and impurity has to have a special form in order to preserve the integrability. Andrei and Johannesson [8] (see also Refs. [9,10]) incorporated a magnetic impurity of arbitrary spin into the isotropic spin-$\frac{1}{2}$ Heisenberg chain without spoiling the integrability.

In recent papers we succeeded in constructing 1D integrable correlated electron lattice models with a magnetic impurity *via* the quantum inverse scattering method [4–7]. Several combinations of hosts, *e.g.* two supersymmetric variants of the non-degenerate $t-J$ model and the Hubbard model, and impurities, *e.g.* exchange and intermediate valence impurities, have been considered. The scattering matrix of the electrons in the host and the scattering matrix of electrons with the impurity have to satisfy the triangular Yang-Baxter relation. This is the necessary and sufficient condition for the integrability, which imposes restrictions on the impurity. The overall picture emerging from this study is: (i) Correlations in the host strongly couple to the charge sector of the impurity without affecting the spin sector. The screening of the impurity spin is then unchanged with respect to free electrons. (ii) The correlations drive the impurity away from integer valence into the mixed valence region with the concomitant increase of the Kondo temperature. A fraction of itinerant electron (hole) is localized at the impurity site. (iii) The impurity is placed on a link of the chain and interacts with both

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neighboring lattice sites. Hence, the impurity interacts with both partial waves, *i.e.* with states of even and odd parity with respect to the impurity site, in contrast with the situation in a noninteracting host, where the coupling is only with even parity (s-wave) conduction states. (iv) The coupling parameter between impurity and host is the impurity rapidity, p_0, which on the one hand determines the Kondo temperature and on the other hand introduces a chirality (right-left asymmetry) into the chain. (v) The impurity is a forward scatterer only and hence does not give rise to boundstates into the system.

In this paper we extend this investigation to an impurity with orbital degrees of freedom. The possibly simplest impurity with combined spin and orbital degeneracy coupled *via* a large spin-orbit interaction is the degenerate Anderson impurity in the $U \to \infty$ limit. This impurity model is an adequate representation for Ce and Yb ions in mixed valent states. The model for the impurity in a noninteracting host has been solved long ago *via* nested Bethe *Ansätze* [11,12]. The results are in good quantitative agreement with experimental data for modestly heavy fermion (Yb and Ce) alloys and compounds [12,13]. The ligand atoms in heavy fermion compounds frequently involve correlated states such as p and d orbitals, so that the effects of these correlations on the impurity should be studied. A similar situation occurs with impurities embedded into high T_c cuprates.

All cases of magnetic impurities in correlated hosts studied so far do not involve orbital degeneracy, with the exception of a recent contribution on the Coqblin-Schrieffer impurity of spin S embedded into the SU($2S$+2)-invariant (the spin and the charge play identical roles) supersymmetric $t-J$ model [14]. The host considered in the present paper is the graded $(F^N B)$ degenerate supersymmetric $t-J$ model, in which the $N = 2S+1$ spin degrees of freedom have fermion (F) symmetries, while the charge is a boson (B). This model is the straightforward extension of the standard spin-1/2 supersymmetric $t-J$ model [15–18] to N spin components and its diagonalization using N nested Bethe *Ansätze* can be found in references [19,20].

The rest of the paper is organized as follows. The sufficient condition for the integrability is the factorization of the many-electron scattering matrix into two-particle scattering matrices (Yang-Baxter triangular relation), which imposes conditions on the scattering matrices between itinerant electrons and of electrons with the impurity. The model is then defined by the scattering matrices *via* the Quantum Inverse Scattering Method [21]. The vertex weights, the monodromy matrix, the diagonalization of the transfer matrix and the discrete Bethe ansatz equations are introduced in Section 2. These equations define the lattice model and determine the properties of the impurity. In Section 2 the interaction Hamiltonian between the impurity and the correlated itinerant electrons is constructed for the continuum limit. In Section 3 we classify the solutions of the Bethe ansatz equations according to the string hypothesis and derive the thermodynamic Bethe ansatz equations. The groundstate and low-T thermodynamic properties of the impurity are studied in Section 4. The impurity can absorb (and release again) one conduction electron and form an effective spin S. The impurity has therefore intermediate valence character, which can be changed as a function of a model parameter (impurity rapidity). In the integer valent limit we relate our model to the Coqblin-Schrieffer impurity. A summary with concluding remarks follows in Section 5.

2 Transfer matrix and the Bethe ansatz equations

We begin this section by briefly restating the results for the supersymmetric $t-J$ model we need here. Then we introduce the impurity scattering matrix, construct the monodromy matrix and derive the discrete Bethe ansatz equations for the lattice model. We present the general form of the lattice interaction Hamiltonian between the impurity and the correlated itinerant electrons. Finally, we explicitly construct the Hamiltonian in the continuum limit (Luttinger liquid limit).

2.1 Degenerate supersymmetric t − J model

The one-dimensional $t-J$ model is defined by the Hamiltonian [19]

$$H_0 = -t\sum_{is} P\big(c_{is}^\dagger c_{i+1s} + c_{i+1s}^\dagger c_{is}\big)P + J\sum_{iss'}\big(c_{is}^\dagger c_{is'} c_{i+1s'}^\dagger c_{i+1s} - n_{is}n_{i+1s'}\big), \quad (1)$$

where $c_{is}^\dagger$ creates an electron of spin component s ($s \le S$) at the site i, P is a projector that excludes the multiple occupancy of every site, and $n_{is} = c_{is}^\dagger c_{is}$ is the number operator for site i and spin component s. Here J is the exchange coupling (assumed antiferromagnetic) and without loss of generality t can be equated to 1. The generalized spin S can be thought of as composed of spin and orbital degrees of freedom.

Model (1) is only integrable for $J = t$, *i.e.* at the supersymmetric point [15–17]. The scattering matrix for two electrons with wavenumbers k_1 and k_2 then takes the form [17]

$$\hat{X}(k_1,k_2) = \frac{(p_1-p_2)\hat{I} + i\hat{P}}{p_1-p_2+i}, \quad (2)$$

where $p = \frac{1}{2}\cot(k/2)$, $\hat{I} = \delta_{s_1 s_1'}\delta_{s_2 s_2'}$ and $\hat{P} = \delta_{s_1' s_2}\delta_{s_2' s_1}$ are the identity and permutation operators for the spin indices, respectively. Here unprimed (primed) spin indices refer to states before (after) scattering. It is easy to verify that the two-electron scattering matrix satisfies the Yang-Baxter triangular relation

$$X_{s_2 s_2'}^{s_1 s_1'}(p_1-p_2)X_{s_3 s_3'}^{s_1' s_1''}(p_1-p_3)X_{s_3' s_3''}^{s_2' s_2''}(p_2-p_3) = X_{s_3 s_3'}^{s_2 s_2'}(p_2-p_3)X_{s_3' s_3''}^{s_1 s_1'}(p_1-p_3)X_{s_2' s_2''}^{s_1' s_1''}(p_1-p_2), \quad (3)$$

where repeated indices are summed over. Relation (3), in addition to the excluded multiple occupancy of sites, are necessary and sufficient conditions for the integrability of equation (1). The energy for a particle of wavenumber k is [17]

$$E = -2\cos(k) = -2 + 2\frac{1/2}{p^2 + 1/4}\cdot \tag{4}$$

2.2 Impurity scattering matrix

We introduce the impurity *via* its scattering matrix with the itinerant electrons. If the integrability of the model is to be preserved, the impurity scattering matrix $\hat{S}$ has to satisfy the following triangular Yang-Baxter relation [12,22]

$$X^{s_1 s_1'}_{s_2 s_2'}(p_1 - p_2) S^{s_1' s_1''}_{M M'}(p_1 - p_0) S^{s_2' s_2''}_{M' M''}(p_2 - p_0) = S^{s_2 s_2'}_{M M'}(p_2 - p_0) S^{s_1 s_1'}_{M' M''}(p_1 - p_0) X^{s_1' s_1''}_{s_2' s_2''}(p_1 - p_2), \tag{5}$$

where the sum over repeated indices is implicit. The index M refers to the spin component of the magnetic impurity. There is not a unique impurity scattering matrix satisfying (5), *i.e.* in principle, more than one impurity form could be constructed without destroying the integrability of the supersymmetric $t-J$ model.

In this paper we consider the impurity scattering matrix [11,12]

$$S^{\sigma\sigma'}_{M M'}(p - p_0) = \delta_{\sigma\sigma'}\delta_{M M'}\frac{p - p_0 + i/2}{p - p_0 - i/2}, \tag{6}$$

where again the unprimed (primed) indices refer to the incoming (outgoing) states and necessarily $|M| \leq S$. The scattering matrix is diagonal in the spin indices, *i.e.* the impurity couples to the charge sector *via* resonant scattering. The impurity is then capable of temporarily absorbing one conduction electron to form an effective spin S, *i.e.* the groundstate is a linear superposition of two different electronic configurations, namely the one without electrons and the one with one localized electron. This is characteristic of intermediate valence systems [11,12]. Here p_0 is the parameter that controls the degree of "valence admixture". Note that both, equations (2, 6), are unitary.

Note that in equation (6) we could have chosen an arbitrary imaginary part for the resonance phase shift. This situation was studied in reference [3] for $p_0 = 0$ and $S = \frac{1}{2}$, which corresponds to a nonmagnetic impurity since the double occupation of the impurity site by electrons is not forbidden. The present choice of phase shift, however, refers to the well-established physical situation of intermediate valent Ce and Yb ions, *i.e.* to a magnetic impurity.

2.3 Monodromy matrix and Bethe ansatz equations

The monodromy matrix [12,21,22] is defined as

$$L^{\{s_1' \ldots s_N' M'\}\tau'}_{\{s_1 \ldots s_N M\}\tau}(\alpha; \alpha_1, \ldots, \alpha_{N+1}) = X^{\tau' \mu_1}_{s_1' s_1}(\alpha_1 - \alpha) X^{\mu_1 \mu_2}_{s_2' s_2}(\alpha_2 - \alpha) \ldots \ldots X^{\mu_{N-1}\mu_N}_{s_N' s_N}(\alpha_N - \alpha) S^{\mu_N \tau}_{M' M}(\alpha_{N+1} - \alpha), \tag{7}$$

with the implicit summation over all the μ_j indices. With respect to the indices τ and τ' the monodromy matrix forms a 2×2 matrix, which we will denote $\hat{L}^{\tau'}_{\tau}(\alpha)$ omitting the spin indices and the parameters α_j.

From the Yang-Baxter relations it follows that the monodromy matrix satisfies the identity [12,21,22]

$$X^{\tau_1 \tau_1'}_{\tau_2 \tau_2'}(\alpha - \alpha')\hat{L}^{\tau_1'}_{\tau_3}(\alpha')\hat{L}^{\tau_2'}_{\tau_3'}(\alpha) = \hat{L}^{\tau_2}_{\tau_2'}(\alpha)\hat{L}^{\tau_1}_{\tau_1'}(\alpha') X^{\tau_1' \tau_3}_{\tau_2' \tau_3'}(\alpha - \alpha'), \tag{8}$$

where the sum over repeated indices is implicit. With the help of this identity it can be shown that transfer matrices, defined as $\hat{T}(\alpha) = \sum_\tau \hat{L}^{\tau}_{\tau}(\alpha)$, at different α values (spectral parameter) commute and can all be diagonalized simultaneously.

Consider now N_e itinerant electrons and the impurity in a box of N_a sites with periodic boundary conditions. Periodic boundary conditions imposed on a given electron means that it has to interchange position with all other electrons. Each shifting through (permutation) involves a two-particle scattering matrix, such that when the particle is back at the original position we obtained an operator that consists of a product of $(N_e - 1)$ electron-electron scattering matrices, $\hat{X}$, and one scattering matrix due to the impurity, $\hat{S}$, *i.e.*

$$\hat{T}_j(k_j) = \hat{X}^{-1}_{j,j+1}(p_j - p_{j+1}) \ldots \hat{X}^{-1}_{j,N}(p_j - p_N) \times \hat{S}^{-1}_j(p_j - p_0)\hat{X}^{-1}_{j,1}(p_j - p_1) \ldots \hat{X}^{-1}_{j,j-1}(p_j - p_{j-1}). \tag{9}$$

The periodic boundary condition for each electron gives rise to one such operator, *i.e.* $j = 1, \ldots, N_e$, and the N_e operators have to be diagonalized simultaneously. The corresponding eigenvalues are

$$\exp(ik_j N_a) = \left[\frac{p_j + i/2}{p_j - i/2}\right]^{N_a}. \tag{10}$$

With $\alpha_{N+1} = p_0$, $\alpha_l = p_l$ for $l = 1, \ldots, N_e$ and $\alpha = p_j$, $j = 1, \ldots, N_e$, equations (9) are just the trace over the monodromy matrix, which as argued above can all be diagonalized simultaneously.

The procedure to diagonalize the transfer matrices is standard and will not be repeated here. The starting point is the totally spin-polarized state and other wavefunctions are constructed by flipping spins in the system. The Bethe *Ansatz* equations are the conditions under which a wavefunction corresponding to a given Young tableau

is an eigenstate of the transfer matrix eigenvalue problem. This eigenvalue problem has been solved by Sutherland [23] for an arbitrary Young tableau by means of a sequence of additional $(N-1)$ nested Bethe *Ansätze*. Each Bethe *Ansatz* leads to a new eigenvalue problem with the number of spin components reduced by one and gives rise to a set of rapidities. This procedure is repeated until all internal degrees of freedom are eliminated. As a result, N sets of rapidities $\{\xi_\alpha^{(l)}\}$, $l=0,\cdots,N-1$, are obtained, which are selfconsistently determined by the Bethe *Ansatz* equations [19,20]. The set for $l=0$ corresponds to the charge rapidities, $\xi_\alpha^{(0)} = p_\alpha = \frac{1}{2}\cot(k_\alpha/2)$, where $\{k_\alpha\}$ are the wavenumbers of the particles, while the other sets are associated with the spin degrees of freedom. All rapidities within a given set have to be different to ensure linearly independent wavefunctions. The discrete Bethe *Ansatz* equations for the degenerate $t-J$ model with impurity are [19]

$$\frac{p_j-p_0+\frac{1}{2}i}{p_j-p_0-\frac{1}{2}i}\left[\frac{p_j+\frac{1}{2}i}{p_j-\frac{1}{2}i}\right]^{N_a} = \prod_{\beta=1}^{M_1}\frac{p_j-\xi_\beta^{(1)}+\frac{1}{2}i}{p_j-\xi_\beta^{(1)}-\frac{1}{2}i},$$
$$j=1,\ldots,N_e, \tag{11}$$

$$\prod_{\beta=1}^{M_{l-1}}\frac{\xi_\alpha^{(l)}-\xi_\beta^{(l-1)}+\frac{1}{2}i}{\xi_\alpha^{(l)}-\xi_\beta^{(l-1)}-\frac{1}{2}i}\prod_{\beta=1}^{M_{l+1}}\frac{\xi_\alpha^{(l)}-\xi_\beta^{(l+1)}+\frac{1}{2}i}{\xi_\alpha^{(l)}-\xi_\beta^{(l+1)}-\frac{1}{2}i} = -\prod_{\beta=1}^{M_l}\frac{\xi_\alpha^{(l)}-\xi_\beta^{(l)}+i}{\xi_\alpha^{(l)}-\xi_\beta^{(l)}-i}$$
$$\beta=1,\ldots,M^{(l)},\quad l=1,\cdots,N-1,\ M_0\equiv N_e,$$
$$M_N\equiv 0, \tag{12}$$

where M_l is the number of rapidities in the set $\{\xi_\alpha^{(l)}\}$. If n_{S-m} denotes the number of electrons with spin component m and $M_{i+1}=M_i-n_i$, then necessarily $N_e\equiv M_0\geq M_1\geq M_2\geq\cdots\geq M_{N-1}\geq M_N\equiv 0$. This solution corresponds to the Young tableau $(M_0-M_1, M_1-M_2,\cdots,M_{N-2}-M_{N-1}, M_{N-1}-M_N)$. The Bethe *Ansatz* equations are only a basis of states within this subspace. The energy eigenvalues of the Hamiltonian (1) and the magnetization are given by

$$E=-2N_e+2\sum_{\alpha=1}^{M_0}\frac{\frac{1}{2}}{(\xi_\alpha^{(0)})^2+\frac{1}{4}}$$
$$S_z=\tfrac{1}{2}(N-1)N_e-\sum_{l=1}^{N-1}M_l. \tag{13}$$

The first factor on the left-hand side in equation (11) arises from the impurity. The remaining factors correspond to the supersymmetric $t-J$ host.

2.4 Impurity Hamiltonian

Our model is defined by the scattering matrices, (2) and (6), *via* the Quantum Inverse scattering Method. By construction of the transfer matrix the impurity spin can be assumed on a given link and interacting only with the sites joined by the link. The Hamiltonian and higher conserved currents, which describe the interaction between the impurity and the itinerant electrons can be obtained by differentiating the logarithm of the transfer matrix $\hat{T}(\alpha)$ with respect to the spectral parameter α at the point $\alpha=0$. The first derivative determines the Hamiltonian of the lattice interacting with the impurity, $H_{t-J,imp}=H_0+H_{imp}$, where H_0 is given by equation (1). H_0 can conveniently be written in terms of Hubbard operators as $H_0=\sum_{n=1}^{N_a-1}H_{n,n+1}$, where [18]

$$H_{n,n+1}=-\sum_{s,s'}(X_n^{s,0}X_{n+1}^{0,s}+X_n^{0,s}X_{n+1}^{s,0}-X_n^{s,s'}X_{n+1}^{s',s}+X_n^{0,0}X_{n+1}^{0,0}). \tag{14}$$

Situating the impurity on the link joining the sites N_a and 1, we obtain for the impurity Hamiltonian

$$H_{imp}=h_1(p_0)(H_{N_a,imp}+H_{imp,1}+\{H_{N_a,imp},H_{imp,1}\})+h_2(p_0)H_{N_a,1}+ih_3(p_0)[(H_{N_a,imp}+H_{imp,1}),H_{N_a,1}], \tag{15}$$

where

$$H_{N_a,imp}=-\sum_s(X_{N_a}^{s,0}X_{imp}^{0,s}+X_{N_a}^{0,s}X_{imp}^{s,0}-X_{N_a}^{s,s}X_{imp}^{s,s}+X_{N_a}^{0,0}X_{imp}^{0,0})$$
$$H_{imp,1}=-\sum_s X_{imp}^{s,0}X_1^{0,s}+X_{imp}^{0,s}X_1^{s,0}-X_{imp}^{s,s}X_1^{s,s}+X_{imp}^{0,0}X_1^{0,0}). \tag{16}$$

Here $X^{a,b}=|a\rangle\langle b|$ are the Hubbard operators ($a,b=0,s$ with $|s|\leq S$) and the square (curly) bracket in (15) denotes commutator (anticommutator). Note that both host and impurity states are restricted to only one particle (or hole) at each site. $h_1(p_0)$ and $h_2(p_0)$ are even functions of p_0 and $h_3(p_0)$ is an odd function of p_0. The impurity Hamiltonian breaks the parity (P) and the time reversal (T) symmetries separately, but, of course, PT is conserved. The sign of the parameter p_0 is only important if finite size effects are considered, since it gives rise to a mesoscopic momentum.

To gain some insight into the physics of the impurity and its interaction with the conduction electrons, it is instructive to derive the Hamiltonian in the continuum limit of the model, *i.e.* in the limit where the lattice constant tends to zero. In the continuum limit we can linearize the kinetic energy in the momentum around the Fermi level and restrict ourselves to low-energy excitations. Assume the two Fermi points are given by $\pm k_{FS}$ related to $\pm p_{FS}$ by $p_{FS}=\frac{1}{2}\cot(k_{FS}/2)$. Denoting $v=[2\sin(k_{FS}/2)]^{-2}$ the group velocity of the electrons, the resonance phase shift corresponds to an effective impurity Hamiltonian

$$H_{imp}=\epsilon\sum_s|s\rangle\langle s|+V\sum_s\int dx\,\delta(x)\left[c_s^\dagger(x)|0\rangle\langle s|+|s\rangle\langle 0|c_s(x)\right], \tag{17}$$

where the hybridization V has to be equated to $v^{-1/2}$. Here ϵ is related to p_0 and represents the energy difference between the two electronic configuration relative to the Fermi level. The impurity states are denoted by the bra and ket. The two partial waves in 1D, *e.g.* forward and backward moving electrons, can be transformed into even and odd parity states about the impurity site. Odd parity states do not interact directly with the impurity in the continuum limit (contact potential), and affect the impurity only through the interactions in the host (Luttinger liquid).

There is a fundamental difference between the impurity embedded in the noninteracting gas of electrons [11,12] and in the correlated electron gas discussed here. In the former case the properties of the impurity are determined as a function of two parameters, namely, ϵ and V, so that charge and spin-fluctuations can occur on different energy scales. The impurity in the correlated gas of electrons has only one free parameter, p_0, while the second parameter is fixed by the condition of integrability. The charge fluctuations and the Kondo screening occur on the same energy scale in the present case.

3 Thermodynamics

In this section we first classify the possible states of the system according to the string hypothesis, then we derive the thermodynamic Bethe ansatz equations, and finally we briefly discuss the $T \to 0$ (groundstate) limit of the thermodynamic equations.

3.1 Classification of states

Each eigenstate of the system is specified by sets of rapidities representing a solution of the discrete Bethe *Ansatz* equations (11, 12). The rapidities have in general complex values and in the thermodynamic limit (large N_a, N_e and M_l, keeping the ratios constant), they can be classified according to:

(i) real charge rapidities, belonging to the set $\{\xi_\alpha^{(0)}\}$, which correspond to unpaired propagating electrons;

(ii) complex spin and charge rapidities, which correspond to boundstates of electrons with different spin components; and

(iii) strings of complex spin rapidities, which represent bound spin states.

The attractive interaction between electrons builds spin complexes of up to N electrons. A complex of n electrons ($n \le N$) is characterized by one real $\xi^{(n-1)}$ rapidity and in general complex $\xi^{(l)}$ rapidities, $l < n-1$, given by

$$\xi_p^{(l)} = \xi^{(n-1)} + \frac{i}{2}p, \quad l \le n-1 \le 2S,$$
$$p = -(n-l-1), -(n-l-3), \cdots, (n-l-1). \quad (18)$$

These spin and charge strings form classes (i) and (ii), which are already present in the groundstate. In class (iii) there is a set of strings of complex spin rapidities for each set of real spin rapidities $\{\xi_\alpha^{(l)}\}$, $l = 1, \cdots, 2S$. A string of length n is given by

$$\xi_{\alpha n}^{(l)\mu} = \Lambda_{\alpha\mu}^{(l)} + \frac{i}{2}\mu, \quad \mu = -(n-1), -(n-3), \cdots, (n-1), \quad (19)$$

where $\Lambda_{\alpha n}^{(l)}$ is a real parameter and α is the running index in each set.

The structure of the solutions of the Bethe ansatz equations is determined by the host (supersymmetric $t-J$ model) [19], rather than by the impurity. However, the classification of states is also similar to that of the spin S Anderson impurity in the $U \to \infty$ limit [24].

We introduce the usual distribution functions for each class of rapidities and their "holes", *i.e.* $\rho^{(l)}(\xi)$ and $\rho_h^{(l)}(\xi)$ for the real $\xi_\alpha^{(l)}$, and $\sigma_n^{(l)}(\Lambda)$ and $\sigma_{n,h}^{(l)}(\Lambda)$ for the $\Lambda_{\alpha n}^{(l)}$. In view of the Fermi statistics obeyed by the rapidities, 'particle' and 'hole' densities are not independent, but coupled by sets of linear integral equations. Fourier transforming the equations we obtain [19]

$$\hat\rho_h^{(l)}(\omega) + \hat\rho^{(l)}(\omega) + \sum_{q=0}^{2S} \hat\rho^{(q)}(\omega) \times \exp\left(-\frac{|\omega|}{2}(l+q-p_{l,q})\right) \frac{\sinh[\omega(p_{l,q}+1)/2]}{\sinh(\omega/2)} + \sum_{n=1}^{\infty} \hat\sigma_n^{(l+1)}(\omega) \exp\left(-n\frac{|\omega|}{2}\right) = \exp\left(-(l+1)\frac{|\omega|}{2}\right) + \frac{1}{N_a}\exp\left(ip_0\omega - (l+1)\frac{|\omega|}{2}\right), \quad (20)$$

$$\hat\sigma_{m,h}^{(l)}(\omega) = \hat\rho^{(l-1)}(\omega)\exp\left(-m\frac{|\omega|}{2}\right) + \sum_{n=1}^{\infty}\left[\hat\sigma_n^{(l-1)}(\omega) + \hat\sigma_n^{(l+1)}(\omega) - 2\cosh\left(\frac{\omega}{2}\right)\hat\sigma_n^{(l)}(\omega)\right] \times \exp\left(-\frac{|\omega|}{2}\max(m,n)\right)\frac{\sinh[\omega\min(m,n)/2]}{\sinh(\omega/2)}. \quad (21)$$

The last set of equations holds for $m = 1, \cdots, \infty$ with $\hat\sigma_m^{(0)}(\omega)$, $\hat\sigma_{m,h}^{(0)}(\omega)$, $\hat\sigma_m^{(N)}(\omega)$, and $\hat\sigma_{m,h}^{(N)}(\omega)$ being identically zero, and $p_{l,q} = \min(l,q) - \delta_{l,q}$. The *hat* denotes a Fourier transform. The term proportional to $1/N_a$ is due to the impurity phase shift. In terms of the densities the energy is given by [19]

$$E = -2N_e + 2N_a \sum_{m=0}^{2S} \int d\xi\ \rho^{(m)}(\xi)\frac{(m+1)/2}{\xi^2 + (m+1)^2/4}. \quad (22)$$

3.2 Thermal equilibrium

The above equations are valid quite generally for all states. In thermal equilibrium the population of the energy levels

is determined by the thermal dressed energy for each class of rapidities defined as

$$\rho_h^{(l)}/\rho^{(l)} = \exp(\varepsilon^{(l)}/T), \quad \sigma_{n,h}^{(l)}/\sigma_n^{(l)} = \exp(\varphi_n^{(l)}/T) = \eta_n^{(l)}, \tag{23}$$

which satisfy the following integral equations [19]

$$\begin{aligned}\varepsilon^{(2S)}(\xi) = &-2-\mu+2\pi H_1(\xi)\\ &+T\sum_{q=0}^{2S-1} F_{q+1}\star\ln[1+\exp(-\varepsilon^{(q)}/T)]\\ &+TH_{2S}\star\ln[1+\exp(\varepsilon^{(2S)}/T)],\end{aligned} \tag{24}$$

$$\begin{aligned}\ln[1+\exp(\varepsilon^{(m)}/T)] = &\frac{2\pi}{T}F_{2S-m}(\xi)\\ &+F_{m+1}\star\ln[1+\exp(\varepsilon^{(2S)}/T)]\\ &+\sum_{q=0}^{2S-1}\Big\{G^{cosh}_{m+1,q+1}\star\ln[1+\exp(-\varepsilon^{(q)}/T)]\\ &-G_{m+1,q+1}\star\ln[1+\eta_1^{(q+1)}]\Big\},\end{aligned} \tag{25}$$

$$\begin{aligned}\varphi_n^{(m)}(\Lambda) = &\,\delta_{n,1}TG_0\star\ln[1+\exp(-\varepsilon^{(m-1)}/T)]\\ &+TG_0\star\ln\left\{\frac{[1+\eta_{n+1}^{(m)}][1+\eta_{n-1}^{(m)}]}{[1+(\eta_n^{(m+1)})^{-1}][1+(\eta_n^{(m-1)})^{-1}]}\right\},\end{aligned} \tag{26}$$

where the last equation holds for $n = 1, ..., \infty$, the *star* denotes convolution, μ is the chemical potential, $\eta_0^{(n)} \equiv 0$, $\eta_n^{(0)} = \eta_n^{(N)} \equiv \infty$ and the integration kernels are the Fourier transform of

$$\hat{F}_m(\omega) = \frac{\sinh(m\omega/2)}{\sinh(N\omega/2)},$$

$$\hat{H}_m(\omega) = \exp(-\tfrac{1}{2}|\omega|)\frac{\sinh(m\omega/2)}{\sinh(N\omega/2)},$$

$$\hat{G}_{l,q}(\omega) = \frac{\sinh(\omega\min(l,q)/2)\sinh[\omega(N-\max(l,q))/2]}{\sinh(N\omega/2)\sinh(\omega/2)},$$

$$\hat{G}^{cosh}_{l,q}(\omega) = 2\cosh(\omega/2)\hat{G}_{l,q}(\omega),$$

$$\hat{G}_0(\omega) = [2\cosh(\omega/2)]^{-1}. \tag{27}$$

In order to be completely defined, equations (26) require asymptotic conditions for the $\varphi_n^{(l)}$ as n tends to infinity. These boundary conditions are determined by the splitting scheme of the $(2S+1)$-fold multiplet, *i.e.* the Zeeman and crystalline field energies. For instance for a pure Zeeman splitting we obtain

$$\lim_{n\to\infty}\frac{1}{n}\varphi_n^{(l)}(\Lambda) = H \geq 0. \tag{28}$$

The equilibrium free energy of the system is the sum of the free energies of the host and the impurity

$$\begin{aligned}\frac{F_{tJ}}{N_a} = &-\frac{T}{\pi}\sum_{m=0}^{2S}\int d\xi\frac{\frac{1}{2}(m+1)}{\xi^2+\frac{1}{4}(m+1)^2}\\ &\times\ln[1+\exp(-\varepsilon^{(m)}(\xi)/T)],\\ F_{imp} = &-\frac{T}{\pi}\sum_{m=0}^{2S}\int d\xi\frac{\frac{1}{2}(m+1)}{(\xi-p_0)^2+\frac{1}{4}(m+1)^2}\\ &\times\ln[1+\exp(-\varepsilon^{(m)}(\xi)/T)].\end{aligned} \tag{29}$$

The expressions for the $t-J$ model and the impurity are analogous, except for the shift of the argument by p_0. This is evident in view of equation (11).

3.3 Groundstate equations

The groundstate integral equations are obtained from equations (24, 25, 26) in the limit $T \to 0$. From equations (26, 28) it follows that the energy potentials $\varphi_n^{(l)}$ are positive over the entire Λ range for all l and $n \geq 1$. Hence, as $T \to 0$ the spin string states are not occupied, and only the $\varepsilon^{(m)}$ bands can be populated. The zeroes of the dressed energies define the Fermi surface (integration limits), $\varepsilon^{(m)}(\pm B_m) = 0$ and $\varepsilon^{(2S)}(\pm Q) = 0$, for spinons and charges, respectively. Empty states (holes) correspond to a positive potential, while a negative dressed energy refers to occupied states (particles). The integration limits are functions of μ and H.

In the limit $T \to 0$ the integral equations satisfied by the dressed energies and densities are of the general form

$$\begin{aligned}&X^{(2S)}(\zeta) - \int_{-Q}^{Q} d\xi'\ X^{(2S)}(\xi')H_{2S}(\xi-\xi')\\ &\quad+\sum_{q=0}^{2S-1}\int_{|\xi'|>B_q} d\xi'\ X^{(q)}(\xi')F_{q+1}(\xi-\xi') = Z_{2S}(\xi),\\ &X^{(m)}(\xi) + \sum_{q=0}^{2S-1}\int_{|\xi'|>B_q} d\xi'\ X^{(q)}(\xi')K_{m+1,q+1}(\xi-\xi')\\ &\quad-\int_{-Q}^{Q} d\xi'\ X^{(2S)}(\xi')F_{m+1}(\xi-\xi') = Z_m(\xi),\end{aligned} \tag{30}$$

where $Z_l(\xi)$ are the driving terms and $K_{l,r}(\xi)$ is the Fourier transform of

$$\hat{K}_{l,r}(\omega) = -\delta_{l,r} + exp(|\omega|/2)\hat{G}_{l,r}(\omega). \tag{31}$$

For $X^{(m)} = \varepsilon^{(m)}$ the driving terms are

$$\begin{aligned}Z_{2S}(\xi) &= -2-\mu+2\pi H_1(\xi),\\ Z_m(\xi) &= 2\pi F_{2S-m}(\xi) - m(N-m)H/2,\end{aligned} \tag{32}$$

where we assumed a Zeeman splitting, while for $X^{(m)} = \rho^{(m)} + \rho_h^{(m)}$ the driving terms are

$$\begin{aligned}Z_{2S}(\xi) &= H_1(\xi) + H_1(\xi-p_0)/N_a,\\ Z_m(\xi) &= F_{2S-m}(\xi) + F_{2S-m}(\xi-p_0)/N_a,\end{aligned} \tag{33}$$

where the terms proportional to N_a^{-1} are the impurity terms.

The energy of the system is given by equation (22) with the integration restricted to the intervals $|\xi| > B_m$. The number of electrons with each spin component is given by ($m = 1, \cdots, N$, and $B_{2S} = Q$)

$$n_m = N_a \sum_{q=N-m}^{2S} \int_{|\xi|>B_q} d\xi \; \rho^{(q)}(\xi). \tag{34}$$

In zero magnetic field we have $B_q = \infty$ for all q, except Q which is finite [19].

4 Properties of the impurity

In this section we present results for the impurity embedded in the supersymmetric $t-J$ lattice. We first consider the groundstate properties in the mixed valent regime, then we discuss the low temperature specific heat and finally the integer valent limit (Coqblin-Schrieffer model). Since the impurity is driven by the host, the supersymmetric $t-J$ model, some impurity properties are then expected to be different from those of the impurity embedded into a noninteracting electron gas.

4.1 Groundstate properties

Equations (20) are linear in the densities and have driving terms arising from the itinerant electrons and from the impurity. Hence, the density functions can be separated into a host and an impurity contribution. The integral equations for the impurity densities are given by equations (30) with the driving terms given by equation (33). In the absence of magnetic and crystalline fields only the $\varepsilon^{(2S)}$ band is populated and $\rho_i^{(2S)}$ is determined by

$$\rho_{i,h}^{(2S)}(\xi) + \rho_i^{(2S)}(\xi) - \int_{-Q}^{Q} d\xi' \rho_{i,h}^{(2S)}(\xi') H_{2S}(\xi - \xi') = H_1(\xi - p_0). \tag{35}$$

This integral equation is of the Fredholm type and differs from the one of the Anderson impurity in a free electron host, which is a Wiener-Hopf equation [11]. Note that the impurity density function is not symmetric in the argument, the asymmetry being introduced by p_0 in the driving term. However, without loss of generality we can symmetrize the driving term by considering the half-sum for $\pm p_0$. This integral equation requires a numerical solution, except in the limits of low electron density ($Q \to \infty$) and low "hole" density ($Q \to 0$). It follows from equation (35) that $0 \le n_{imp} = N \int_{|\xi|\ge Q} d\xi \rho_i^{(2S)}(\xi) \le 1$, *i.e.* the impurity can localize up to one electron. The charge susceptibility of the impurity is defined as

$$\chi_{ch} = \partial n_{imp} \; / \; \partial |p_0|. \tag{36}$$

The impurity groundstate energy, the number of electrons localized by the impurity and the charge susceptibility are shown in Figures 1 and 2 for $N = 6$ (mixed valent Ce impurities) and $N = 8$ (Yb impurities) as a function of p_0 for several bandfillings. Note that for $p_0 = 0$

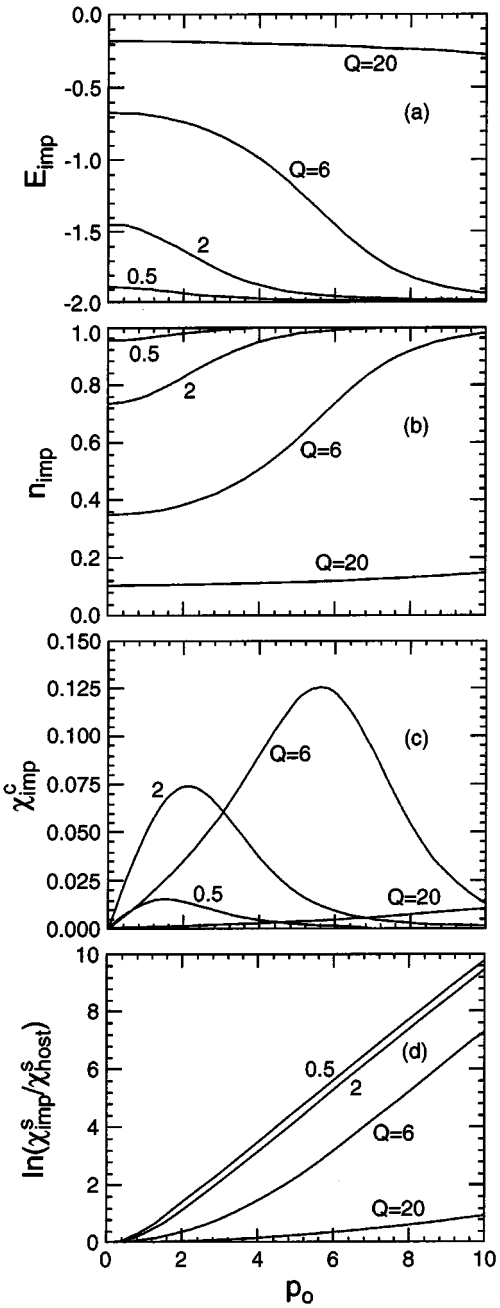

Fig. 1. (a) Impurity groundstate energy, (b) valence (fraction of localized electron at the impurity site), (c) charge susceptibility, and (d) logarithm of the ratio of the spin susceptibilities of the impurity and host (per site) for $N = 6$ as a function of the impurity rapidity p_0. For $p_0 = 0$ the impurity properties are identical to the ones of the host. The four curves refer to different band-fillings of the host, *i.e.* $n = 0.9541$, 0.7351, 0.3501 and 0.1047, respectively. $N = 6$ corresponds to the situation of a mixed valent Ce ion embedded into a metal.

the impurity is just one more link in the chain and its properties are identical to those of the host (per lattice site). Note that the energy and n_{imp} are symmetric functions of p_0, while the impurity charge susceptibility vanishes for $p_0 = 0$. The charge susceptibility is largest when the impurity rapidity is on resonance with the Fermi level. The fraction of localized charge increases with $|p_0|$ when the impurity rapidity enters the continuum of occupied states.

The groundstate zero-field magnetic susceptibility is obtained following standard procedures (see Refs. [11,12, 22]). For a small Zeeman splitting the integration limits B_m for $m = 0, \cdots, 2S-1$ are finite but very large (tending to ∞ as H tends to zero). The two Fermi points of each spinon branch are then well separated and to leading order in the field do not interfere with each other. To obtain the linear response of the impurity to a magnetic field the Fredholm equations for the spinon branches can be reduced to a hierarchical sequence of Wiener-Hopf integral equations. The leading set of equations determines the zero-field susceptibility and is given by

$$\begin{aligned}\rho_{i,h}^{(m)}(\xi) + \rho_i^{(m)}(\xi) &+ \sum_{q=0}^{2S-1} \int_{\xi'>B_q} d\xi'\ \rho_i^{(q)}(\xi') K_{m+1,q+1}(\xi-\xi') \\ &= \int_{-Q}^{Q} d\xi'\ \rho_{i,h}^{(2S)}(\xi') F_{m+1}(\xi-\xi') \\ &+ \frac{1}{2}\Big[F_{2S-m}(\xi-p_0) + F_{2S-m}(\xi+p_0)\Big]. \quad (37)\end{aligned}$$

The density $\rho_{i,h}^{(2S)}$ is also influenced by the magnetic field, through the remaining density distributions, but this effect is of higher order than leading and can be neglected. Hence, in equation (37) we use the solution of equation (35) for $\rho_{i,h}^{(2S)}$ and consider the right-hand side as the driving term of the equations. Since there are two kinds of driving terms (an independent term depending on p_0 and the term involving $\rho_{i,h}^{(2S)}$, the magnetization is the sum of two contributions, the magnetization due to the impurity spin and the magnetization arising from the valence admixture. Expanding the driving terms for large ξ (assuming that $B_m \gg Q$) we obtain that all terms (even those of the host) are proportional to $\exp(-2\pi\xi/N)$, so that asymptotically all the densities are proportional to each other. This proportionality can be used to obtain the ratio of the impurity and host magnetizations in small fields without further calculation

$$\frac{\chi_{imp}}{\chi_{host}} = \frac{\cosh(2\pi|p_0|/N) + \displaystyle\int_{-Q}^{Q} d\xi \cosh(2\pi\xi/N)\rho_{imp,h}^{(2S)}(\xi)}{1 + \displaystyle\int_{-Q}^{Q} d\xi \cosh(2\pi\xi/N)\rho_{host,h}^{(2S)}(\xi)} \cdot \quad (38)$$

The first term represents the Kondo susceptibility, while the second term arises from the spin-fluctuations due to the valence admixture. The absolute value of the host susceptibility has been calculated in reference [20] and is given by

$$\chi_{host} = \frac{N(N^2-1)}{24\pi^2} \times \frac{1 + \displaystyle\int_{-Q}^{Q} d\xi \cosh(2\pi\xi/N)\rho_{host,h}^{(2S)}(\xi)}{1 + (1/2\pi)\displaystyle\int_{-Q}^{Q} d\xi \cosh(2\pi\xi/N)\varepsilon^{(2S)}(\xi)} \cdot \quad (39)$$

The Kondo term of the susceptibility dominates over the contribution due to the valence fluctuations as seen in Figures 1d and 2d, where the logarithm of the ratio (38) of the impurity and host spin susceptibilities is displayed as a function of p_0 for several band fillings. For $p_0 = 0$ this ratio is equal to 1 indicating once more that for $p_0 = 0$ the impurity is just one more link in the chain. The dominance of the Kondo exponential is manifested by the asymptotic straight line of slope $2\pi/N$ approached for sufficiently large p_0. Note that the susceptibility is an even function of p_0. For $Q = 0$ the host is an insulator (no holes) with $N_e = N_a$ and $n_{imp} = 1$, $E_{imp} = 0$ and $\chi_{imp}/\chi_{host} = \cosh(2\pi|p_0|/N)$. In this limit the chain corresponds to the SU(N) Heisenberg antiferromagnet of spin $S = (N-1)/2$.

4.2 Low-temperature specific heat

Since the groundstate of the impurity is a singlet for all parameters, the low-temperature specific heat is proportional to the temperature and can be characterized by its γ_{imp} coefficient. The γ coefficient is calculated from the Sommerfeld expansion of the free energy, equation (29), and the dressed energies, equations (24, 25),

$$\begin{aligned}\gamma_{imp} = \frac{\pi^2}{3}(\partial\varepsilon^{(2S)})^{-1}\Big[H_1(Q-p_0) + H_1(Q+p_0)\Big] &+ \frac{\pi^2}{3}\sum_{q=0}^{2S-1}(\partial\varepsilon^{(q)})^{-1}\Big[F_{2S-q}(B_q-p_0) + F_{2S-q}(B_q+p_0)\Big] \\ &+ 2\int_{-Q}^{Q} d\xi\ H_1(\xi-p_0)\varepsilon_2^{(2S)}(\xi) \\ &- 2\sum_{q=0}^{2S-1}\int_{|\xi|>B_q} d\xi\ F_{2S-q}(\xi-p_0)\varepsilon_2^{(q)}(\xi), \quad (40)\end{aligned}$$

where $\varepsilon^{(q)} = \varepsilon_0^{(q)} + T^2\varepsilon_2^{(q)}$, and $(\partial\varepsilon^{(q)})^{-1} = |d\varepsilon_0^{(q)}/d\xi|_{B_q}^{-1}$ and $(\partial\varepsilon^{(2S)})^{-1} = |d\varepsilon_0^{(2S)}/d\xi|_{Q}^{-1}$. Note that for $p_0 = 0$ the specific heat of the impurity is identical to that of the host, calculated for arbitrary band filling and splitting scheme in reference [25]. The same procedure as in reference [25] will be followed here. The $\varepsilon_2^{(q)}$ can be written as

$$\varepsilon_2^{(q)}(\xi) = (\pi^2/6)\sum_{r=0}^{2S}(\partial\varepsilon^{(r)})^{-1}\ \varphi_r^{(q)}(\xi), \quad (41)$$

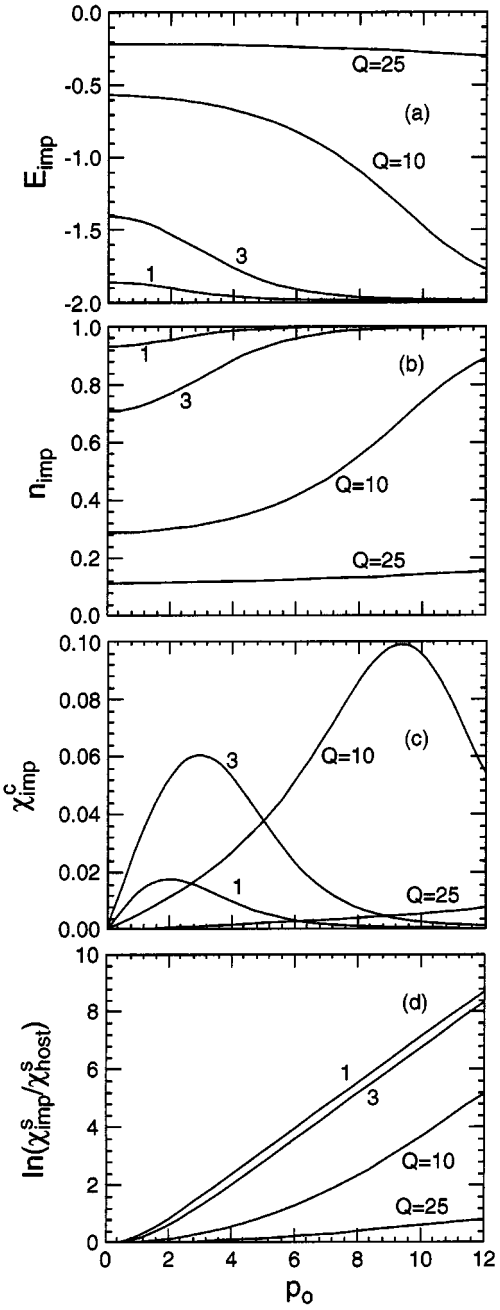

Fig. 2. (a) Impurity groundstate energy, (b) valence (fraction of localized electron at the impurity site), (c) charge susceptibility, and (d) logarithm of the ratio of the spin susceptibilities of the impurity and host (per site) for $N = 8$ as a function of the impurity rapidity p_0. For $p_0 = 0$ the impurity properties are identical to the ones of the host. The four curves refer to different band-fillings of the host, *i.e.* $n = 0.9306$, 0.7092, 0.2882 and 0.1143, respectively. $N = 8$ corresponds to the situation of a mixed valent Yb ion embedded into a metal.

where $\varphi_r^{(q)}(\xi)$ satisfies linearly coupled integral equations of the form (30) with driving terms determined from equations (24, 25). Hence, all contributions to γ are proportional to $\partial\varepsilon_q^{-1}$ for some q.

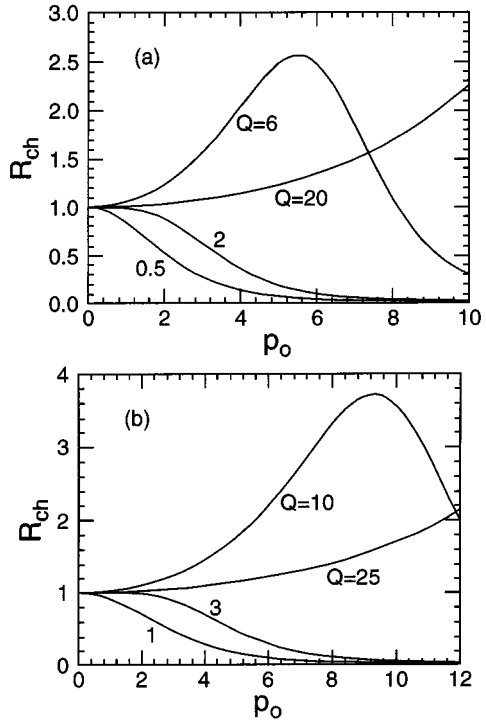

Fig. 3. Enhancement factor R_{ch} for the impurity charge contribution to the specific heat γ_{imp} over the one of the host, $(\rho_{imp}^{(2S)}(Q) + \rho_{imp}^{(2S)}(-Q))/(2\rho_{host}^{(2S)}(Q))$ as a function of p_0 for (a) $N = 6$ and (b) $N = 8$ and the same parameters as in Figures 1 and 2.

Following algebraic manipulations similar to reference [25] we obtain

$$\gamma_{imp} = \frac{\pi}{6} \sum_{q=0}^{2S} \frac{\left[\rho_{imp}^{(q)}(B_q) + \rho_{imp}^{(q)}(-B_q)\right]}{\left[v_q\ \rho_{host}^{(q)}(B_q)\right]}, \qquad (42)$$

where $B_0 = Q$ and v_q is the group velocity of the corresponding excitation branch, given by

$$v_q = \frac{|d\varepsilon_0^{(q)}(\xi)/d\xi|_{B_q}}{2\pi\rho_{host}^{(q)}(B_q)}\,. \qquad (43)$$

Note that the host densities are symmetric functions of the rapidity, while the impurity densities are not, as a consequence of the impurity rapidity. Setting $p_0 = 0$ we obtain the well-known result for the host

$$\gamma_{host}/N_a = (\pi/3) \sum_{q=0}^{2S} \big(v_q\big)^{-1}. \qquad (44)$$

Expressions (42, 44) hold for arbitrary band filling and level splitting.

Expression (42) simplifies in the absence of magnetic and crystalline fields, where only the $\varepsilon^{(2S)}$ band is occupied. The spin contribution arises from the empty bands with Fermi surface at $B_q = \pm\infty$, which all contribute

equally. After some algebra we obtain

$$\gamma_{imp}=\frac{\pi}{6}\frac{\rho_{imp}^{(2S)}(Q)+\rho_{imp}^{(2S)}(-Q)}{v_{2S}\ \rho_{host}^{(2S)}(Q)}+\frac{\pi}{3}\frac{(N-1)}{v_{sw}}\frac{\chi_{imp}}{\chi_{host}}, \quad (45)$$

where v_{sw} is the spinwave velocity, which is inversely proportional to the magnetic susceptibility of the host. The second term in equation (45) is then proportional to the impurity spin susceptibility, while the first term is caused by the charge fluctuations. The interactions in the host make the contribution from the charge sector nonuniversal. The enhancement factor R_{ch} for the impurity charge term over the one of the host, *i.e.* $(\rho_{imp}^{(2S)}(Q)+\rho_{imp}^{(2S)}(-Q))/(2\rho_{host}^{(2S)}(Q))$ is shown in Figure 3 for $N=6$ and $N=8$ and the same parameters as in Figures 1 and 2. For sufficiently large Q (relatively small band-filling) the correlations in the host play a lesser role and R_{ch} has a peak when the impurity rapidity is on resonance with the Fermi level. This is similar to the results obtained for the impurity in an uncorrelated host, where the charge contribution is just the charge susceptibility. For small Q (low density of holons), on the other hand, the correlations in the host suppress this peak. The latter situation is very different from an Anderson impurity embedded into a free electron host.

4.3 Coqblin-Schrieffer limit

The Kondo limit is obtained by suppressing the charge excitations in the system. This integer valent limit is realized for very large $|p_0|$ ($|p_0|\gg Q$), *i.e.* a very small Kondo temperature. Without loss of generality we may choose $p_0>0$. To leading order it is then sufficient to consider only occupied states with $\xi>0$, since the occupied states with $\xi<0$ are far off resonance with the impurity rapidity. The first two terms on the right-hand side of equation (25) can then be approximated as

$$\frac{2\pi}{T}F_{2S-m}(\xi)+F_{m+1}\star\ln[1+\exp(\varepsilon^{(2S)}/T)] \approx 2\frac{\epsilon_F}{T}\sin\left(\pi\frac{m+1}{N}\right)\exp\left(-\frac{2\pi\xi}{N}\right), \quad (46)$$

where we kept only the low-lying excitations, given by large ξ, and ϵ_F is an energy scale of the order of the band half-width.

We now introduce the rescaled variable $\lambda=(2\pi\xi/N)-\ln(\epsilon_F/T)$ and define

$$\Theta_1^{(l+1)}(\lambda)=\exp[-\varepsilon^{(l)}(\xi)/T], \quad \Theta_{n+1}^{(l)}(\lambda)=\eta_n^{(l)}(\xi). \quad (47)$$

The thermodynamic Bethe *Ansatz* equations, (25) and (26), now take the following form

$$\ln[1+(\Theta_1^{(l)})^{-1}]=2\sin(\pi l/N)e^{-\lambda} -\sum_{q=1}^{2S}G_{l,q}\star\ln[1+\Theta_2^{(q)}]+\sum_{q=1}^{2S}G_{l,q}^{cosh}\star\ln[1+\Theta_1^{(q)}], \quad (48)$$

$$\ln\left(\Theta_n^{(l)}\right)=G_0\star\ln\left\{\frac{[1+\Theta_{n+1}^{(l)}][1+\Theta_{n-1}^{(l)}]}{[1+(\Theta_n^{(l+1)})^{-1}][1+(\Theta_n^{(l-1)})^{-1}]}\right\}, \quad (49)$$

where the argument in the integration kernels is now $(N/2\pi)\lambda$ and equation (49) holds for $n\geq 2$ subject to the asymptotic field condition. Suppressing the charge fluctuations the impurity free energy can be cast into the form

$$F_{imp}=-\frac{T}{2\pi}\sum_{l=1}^{2S}\int d\lambda\ \ln[1+\Theta_1^{(l)}(\lambda)] \times\frac{\sin(\pi l/N)}{\cosh[\lambda+\ln(T_K/T)]-\cos(\pi l/N)}, \quad (50)$$

where $T_K=\epsilon_F\exp(-2\pi|p_0|/N)$ is the Kondo temperature. After the variable change from ξ to λ, the only dependence on p_0 is in the free energy.

The coupled integral equations (48), (49) and (50) are those of the Coqblin-Schrieffer model in a noninteracting host [12,22,26]. The approximations made to cast the problem into this form are (i) to suppress the valence fluctuations (charge sector) and (ii) neglect the interference of the two Fermi points for every class of excitations. The latter is justified only if T_K is very small (large $|p_0|$). Then only low-energy excitations play a role and the interacting lattice can be approximated by a Luttinger liquid. As we have already argued, in the continuum limit the impurity interacts only with states of even parity about the impurity site.

The above results are valid for an arbitrary level splitting scheme.

5 Concluding remarks

Impurities play a relevant role in highly correlated electron systems and may alter the properties of the system. In this paper we pursued the exact solution of a degenerate Anderson impurity ($U\to\infty$ limit) embedded into the 1D degenerate supersymmetric $t-J$ model. The impurity model has only one free parameter, p_0, that tunes the properties of the impurity (charge and spin sectors). In contrast, the same impurity in a free electron host has two parameters, such that charge and spin-fluctuations can be varied independently. The impurity is introduced *via* its scattering matrix with the conduction electrons of the host. The scattering matrix in this case is a simple phase shift in the charge sector. The impurity is located on a link and interacts with the itinerant electrons on the two neighboring sites joined by this link. Although the impurity breaks the translational invariance of the system, the condition of integrability requires absence of backward scattering, so that no additional boundstates are introduced into the spectrum. The integrability makes our impurity problem special, but we do not expect that deviations from this condition will qualitatively affect the results.

The model is constructed from the scattering matrices *via* the Quantum Inverse Scattering Method, which

on the one hand defines the Hamiltonian and higher conserved currents, while on the other hand it also yields the discrete Bethe *Ansatz* equations diagonalizing the host with impurity. The states of the system are then classified according to the string hypothesis and the thermodynamic Bethe *Ansatz* equations are derived. We extensively discussed groundstate properties and the low-temperature specific heat for the intermediate valence situation. For the integer-valent limit with a small T_K (large $|p_0|$) we recovered the exact solution of the Coqblin-Schrieffer model.

The impurity can localize up to one conduction electron and is in general in a mixed valent state, consisting of the linear superposition of the empty configuration and the configuration of one localized electron. For $N = 6$ this situation represents a Ce ion and for $N = 8$ to a Yb impurity (if electrons and holes are interchanged). The properties of the impurity strongly depend on the band filling of the host. In particular, for $Q = 0$ the itinerant band is completely filled, and the host corresponds to the SU(N) invariant Heisenberg chain. In this limit the impurity is integer-valent and its magnetic susceptibility is enhanced by the inverse of the Kondo exponential above the value of the susceptibility of the host.

There are two contributions to the magnetic properties at $T = 0$, namely the Kondo magnetization and the magnetization arising from spin-fluctuations due to the valence admixture. The latter is always smaller than the Kondo effect and can be neglected for most purposes. The γ coefficient of the specific heat has contributions arising from the charge fluctuations and from the spin sector. In the absence of spin and crystalline field splittings the impurity contribution from the spin sector is enhanced over the host contribution by a factor given by the ratio of the respective spin susceptibilities. The contribution from the charge sector, on the other hand, is non-universal and depends strongly on the band filling.

To summarize, the model studied in this paper confirms the general results derived previously for other combinations of interacting host and magnetic impurity [4–7,14]. The trends are the same, namely (a) the interactions in the host drive the impurity into a mixed valent state, (b) the Kondo exchange coupling is parametrized by the impurity rapidity, (c) the impurity is on a link of the chain and interacts with both neighboring sites (*i.e.*, with even and odd parity states), and (d) the impurity is a forward scatterer only and does not produce a bound state split off from the continuum. The latter property is clearly a non-universal feature of our model, consequence of the integrability. All other properties are believed to be general and generic.

It has been argued in reference [27] that an integrable magnetic chain embedding an impurity with periodic boundary conditions corresponds to an unstable critical point with non-generic properties. The stable fixed points for the Heisenberg chain with impurity are the unperturbed chain and the chain with a break at the impurity location. It was shown recently [28] that the magnetic behavior of impurities in open and periodic integrable chains are identical for fields and temperatures of the order of or smaller than the Kondo scale. This proof and the similarities of our results with the impurity embedded into the free electron gas are strong indications that our results are generic and not specific to the integrability.

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References

1. F.D.M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980); J. Phys. C **14**, 2585 (1981).
2. D.-H. Lee, J. Toner, Phys. Rev. Lett. **69**, 3378 (1992); A. Furusaki, N. Nagaosa, Phys. Rev. Lett. **72**, 892 (1994); P. Fröjdh, H. Johannesson, Phys. Rev. Lett. **75**, 300 (1995); Phys. Rev. B, **53**, 3211 (1996).
3. G. Bedürftig, F.H.L. Essler, H. Frahm, Phys. Rev. Lett. **77**, 5098 (1996); Nucl. Phys. B **489**, 697 (1997).
4. P. Schlottmann, A.A. Zvyagin, Phys. Rev. B **55**, 5027 (1997).
5. A.A. Zvyagin, P. Schlottmann, J. Phys.-Cond. **9**, 3543 (1997).
6. P. Schlottmann, A.A. Zvyagin, Nucl. Phys. B **501**, 728 (1997).
7. A.A. Zvyagin, P. Schlottmann, Phys. Rev. B **56**, 300 (1997).
8. N. Andrei, H. Johannesson, Phys. Lett. A **100**, 108 (1984).
9. K. Lee, P. Schlottmann, Phys. Rev. B **37**, 379 (1988).
10. P. Schlottmann, J. Phys.-Cond. **3**, 6617 (1991).
11. P. Schlottmann, Z. Phys. B **49**, 109 (1982); *ibid.* **51** 49 (1983).
12. P. Schlottmann, Phys. Rep. **181**, 1 (1989).
13. P. Schlottmann, Z. Phys. B **57**, 23 (1984).
14. P. Schlottmann, J. Phys.-Cond. **10**, 2525 (1998).
15. C.K. Lai, J. Math. Phys. **15**, 167 (1974).
16. B. Sutherland, Phys. Rev. B **12**, 3795 (1975).
17. P. Schlottmann, Phys. Rev. B **36**, 5177 (1987).
18. F.H.L. Essler, V.E. Korepin, Phys. Rev. B **46**, 9147 (1992).
19. P. Schlottmann, J. Phys.-Cond. **4**, 7565 (1992).
20. P. Schlottmann, J. Phys.-Cond. **5**, 313 (1993).
21. V.E. Korepin, N.M. Bogoliubov, A.G. Izergin, *Quantum Inverse Scattering Method* (Cambridge University Press, Cambridge, 1993).
22. A.M. Tsvelick, P.B. Wiegmann, Adv. Phys. **32**, 453 (1983).
23. B. Sutherland, Phys. Rev. Lett. **20**, 98 (1968).
24. P. Schlottmann, Z. Phys. B **54**, 207 (1984).
25. K. Lee, P. Schlottmann, J. Appl. Phys. **79**, 6605 (1996).
26. N. Andrei, K. Furuya, J. Lowenstein, Rev. Mod. Phys. **55**, 331 (1983).
27. E. Sorensen, S. Eggert, I. Affleck, J. Phys. A **26**, 6757 (1993).
28. A.A. Zvyagin, H. Frahm, J. Phys.-Cond. **9**, 9939 (1997); A.A. Zvyagin, Phys. Rev. Lett. **79**, 4641 (1997).